Progression in Calculation

Progression in Calculations

The aim of our calculation policy is to ensure all children leave St. Marie's Catholic Primary School with a secure understanding of the four operations and can confidently use both written and mental calculation strategies in a range of contexts.

This policy sets out the progression of written procedures that the children will use as they progress in their understanding of the four operations.

In order for children to develop a full understanding of the written procedures, they must first have a firm understanding of place value.

It is expected that the majority of pupils will progress through the calculation stages as stated in this policy.

However, children **should not** be made to go onto the next stage if:

1. **They are not ready.**
2. **They are not confident.**

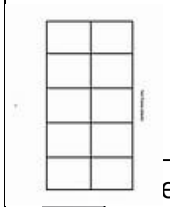
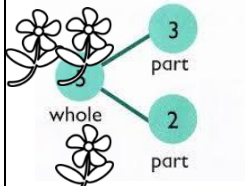
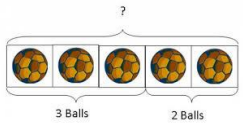
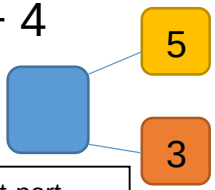
Children who do grasp concepts rapidly should be challenged through sophisticated and diverse problems, before being accelerated through new content.

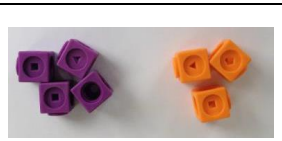
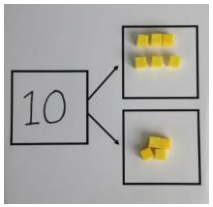
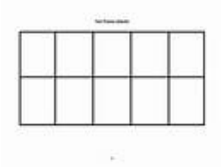
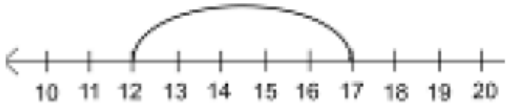
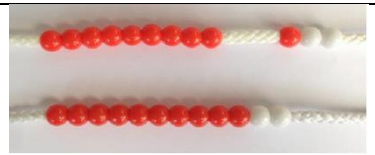
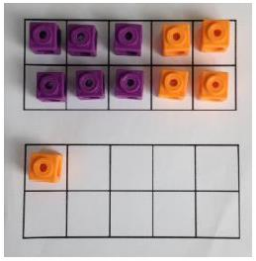
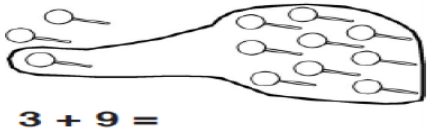
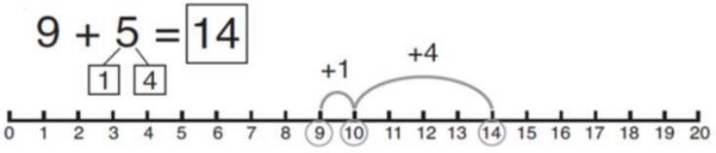
Counting

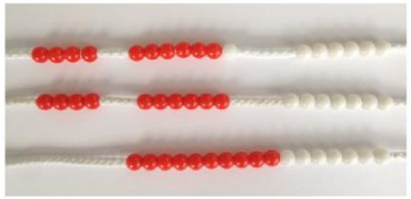
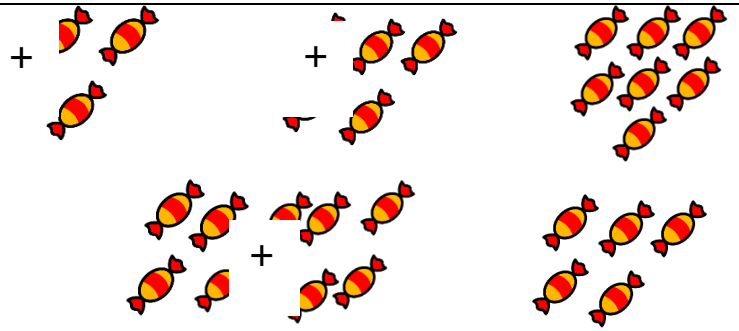
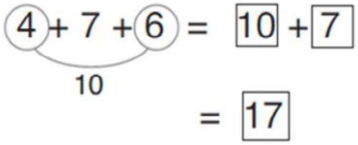
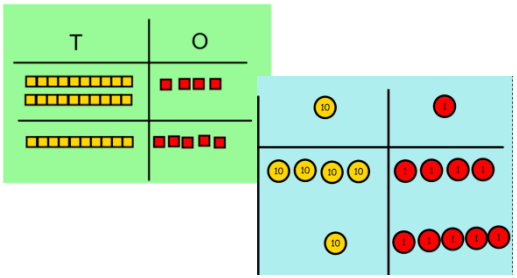
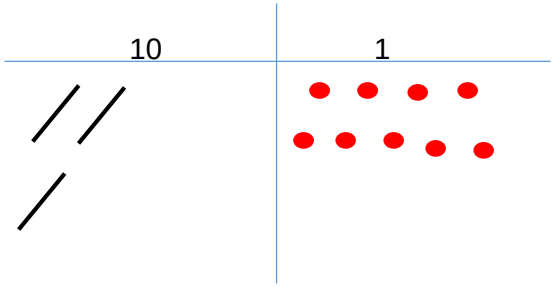
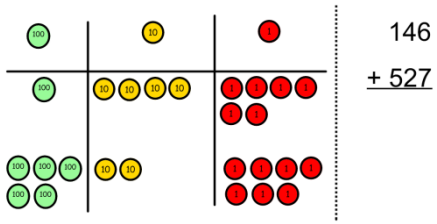
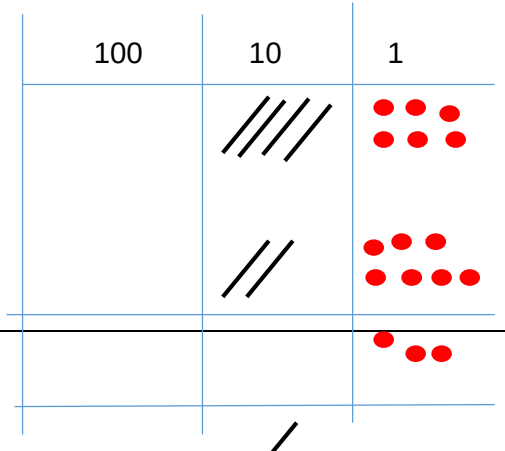
Counting Principles			
Stable Order Principle Understanding that the counting sequence stays consistent. It is always 1, 2, 3, 4, 5, 6, 7, etc. not 1, 2, 4, 5, 8.	**One-to-One Correspondence Principle** Understanding that each object being counted must be given one count and only one count. It is useful in the early stages for children to actually tag each item being counted and to move an item out of the way as it is counted e.g. touch, move and count.	**Cardinality Principle** Understanding that the last count of a group of object represents how many are in the group. Children, who recounts when asked how many sweets are in the group that they just counted, have not understood the cardinality principle.	**Conservation Principle** Understanding that the count for a set group of objects stays the same no matter whether they are spread out or close together.
Abstraction Principle Understanding that the quantity of five large things is the same count as a quantity of five small things. Or the quantity is the same as a mixed group of five small, medium and large things.	**Order Irrelevance Principle** Understanding that the counting of objects can begin with any object in a set and the total will stay the same.	**Movement is Magnitude Principle** Understanding that as you move up the counting sequence, the quantity increases by one and as you move down or backwards, the quantity decreases by one (or by whatever number you are counting by as in skip counting by 10's, the amount goes up by 10 each time).	**Unitizing Principle** Understanding that in our base ten system objects are grouped into tens when the count exceeds 9 and that this is indicated by a 1 in the tens column of a number.

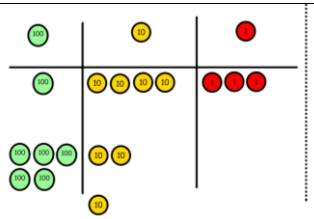
Progression in Calculations

Addition

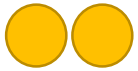
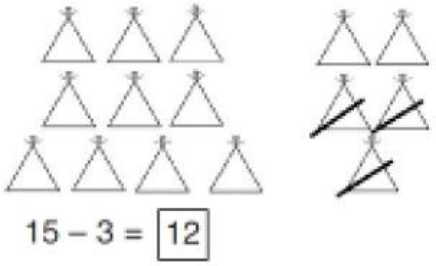
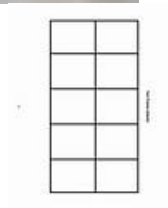
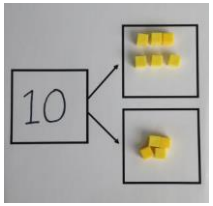
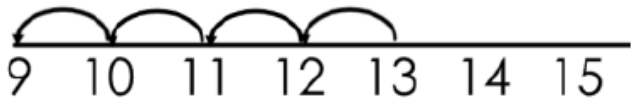
Objective and Strategies	Concrete Examples include structural apparatus such as cubes, counters, 3D shapes or weighing scales as well as contextual objects such as teddies or coins for counting or sorting.	Pictorial Examples include children's own mark making and simple drawings, sketches, number lines and diagrams.	Symbolic Examples include young children's emergent graphics, early number formation, number sentences and written expanded methods such as 'chunking or 'grid' method.
Combining two parts to make a whole: part-whole model	     e cubes to add two numbers together as a group or in a bar.  **Aggregation (count all)**	    Use pictures to add two numbers together as a group or in a bar. 	Introduce = as **'is the same as but looks different to'**. 4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.

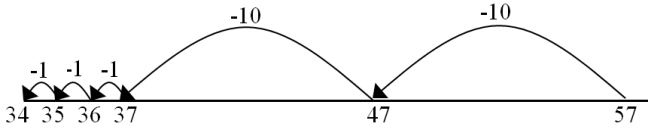
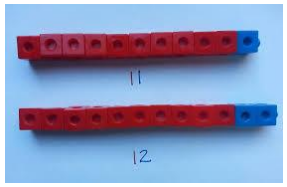
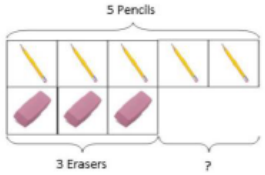
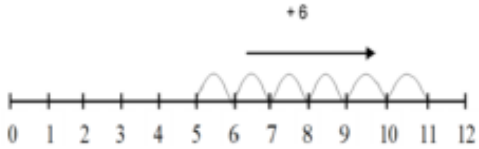
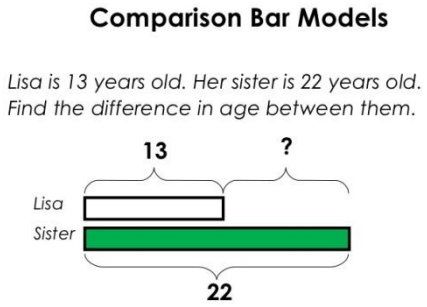
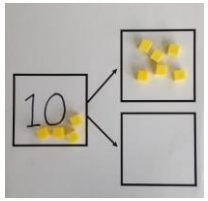
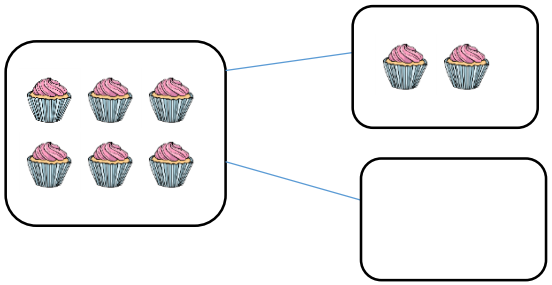
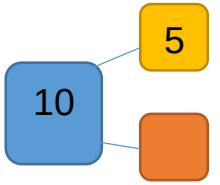
Addition is the inverse of subtraction **Addition is commutative i.e. 5 + 3 = 3 + 5**	    		
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer. (When counting on, the jumps go above the number line)	12 + 5 = 17 Place the larger number in your head and count on the smaller number to find your answer. **Augmentation (count on)**
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10? How many more do I add on now?

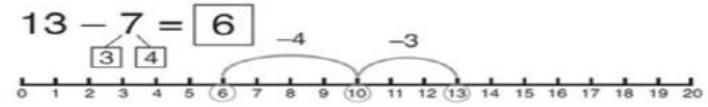
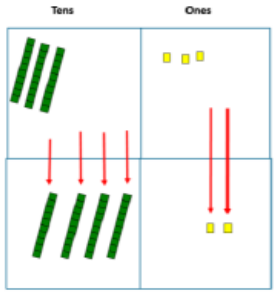
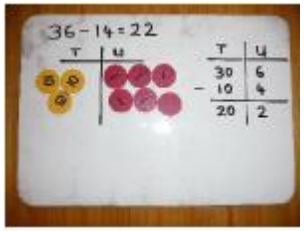
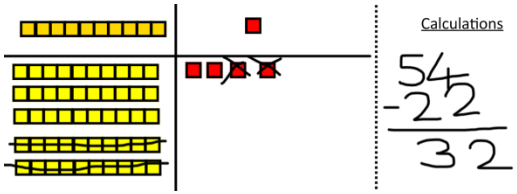
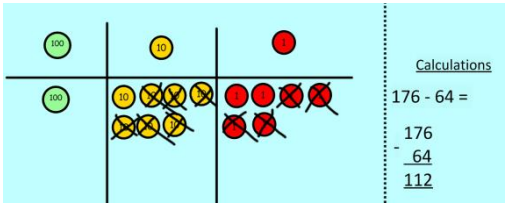
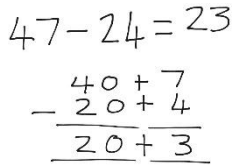
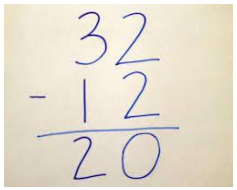
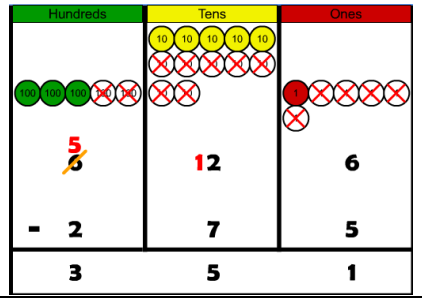
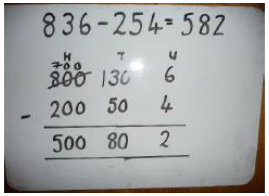
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can use jottings to help them to solve additions. 	Calculation: 20+4 10+5 30+9 24+15=39 20 + 5 40 + 8 60 + 13 = 73
Column method- regrouping	Make both numbers on a place value grid or with Dienes.  146 + 527 Add up the units and exchange 10 ones for one 10.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. 	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 5 40 6 70 1 ~~10~~ Use a diagonal line to show the additonhas been done.

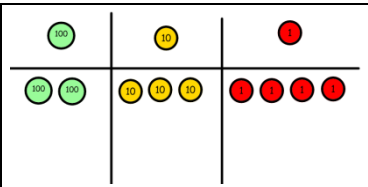
	 146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.		As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 536 + 85 621 1 1 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4   	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Subtraction is the inverse of addition Subtraction is not commutative i.e. 5 - 3 is not the same as 3 - 5	    		
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

	Use counters and move them away from the group as you take them away counting backwards as you go. Use **touch, count, move**. 	 This can progress all the way to counting back using two 2 digit numbers.	
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference Comparison of two amounts – **finding the difference**	 Count on to find the difference. Draw bars to find the difference between 2 numbers. 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches. How many more sandwiches does Hannah have than Helen?
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

Make 10	14 – 5 = 9  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping (see Apendix)	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first and use expanded column subtraction..  	  Draw the Base 10 or place value counters alongside the written calculation to help to show working.	 This will lead to a clear written compact column subtraction. 
Column method with regrouping (see Apendix)	Use Base 10 to start with before moving on to place value counters. Start with one **exchange** before moving onto subtractions with 2 **exchanges**. To help with this use the rhyme:*" If there's more on the floor, go next door."* Make the larger number with the place value counters	 	



Calculations

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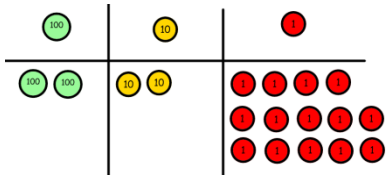
- 88

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

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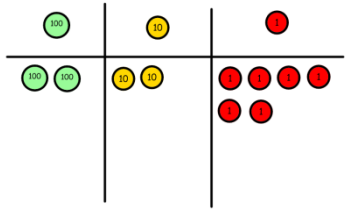
Now I can subtract my ones.



Calculations

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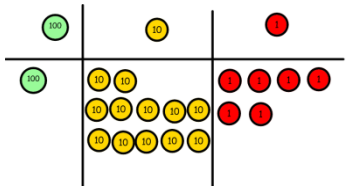
- 88

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Calculations

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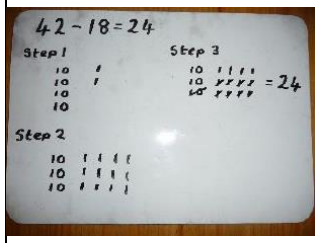
- 88

Now I can take away eight tens and complete my subtraction

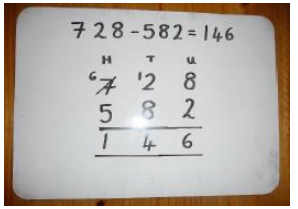
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

When confident, children can find their own way to record the exchange/regrouping.



Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

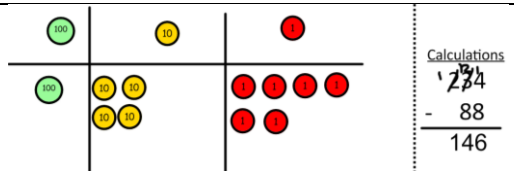
Children can start their formal written method by partitioning the number into clear place value columns – **expanded method.**



Moving forward the children use a more **compact method.**

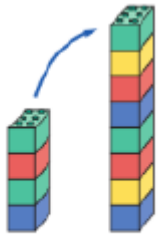
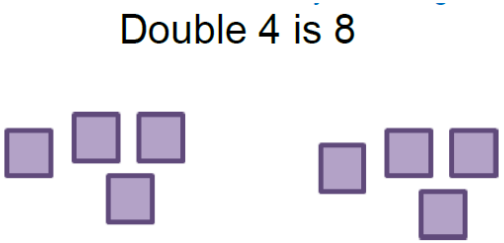
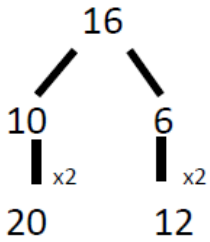
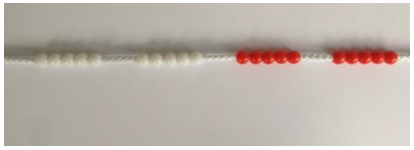
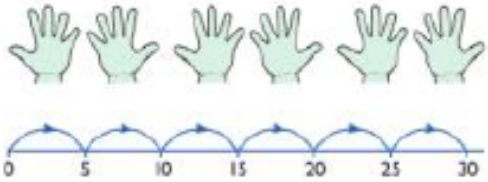
This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} \ 5\ \ 12\ \ \ \ 1 \\ 2\ \not{6}\ \not{3}\ .\ \mathbf{0} \\ -\ \ \ 2\ 6\ .\ 5 \\ \hline 2\ 3\ 6\ .\ 5 \end{array}$$

	 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		
Find the difference	If the numbers are in the same century or are close together on the number line, then finding the difference should be used.		

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract

Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together – **the doubling diamond**.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

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Repeated addition

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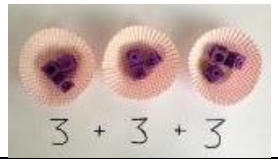
Use different objects to add equal groups.



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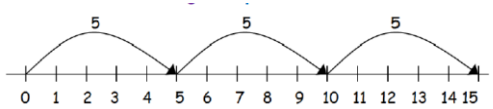
There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?

2 add 2 add 2 equals 6



5 + 5 + 5 = 15

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Write addition sentences to describe objects and pictures.

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Arrays- showing commutative multiplication 6 x 4 = 4 x 6

Use = ‘is the same as but looks different to’.

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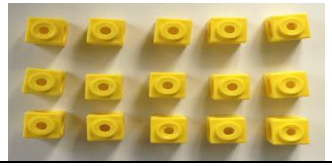
Create arrays using counters/ cubes to show multiplication sentences.



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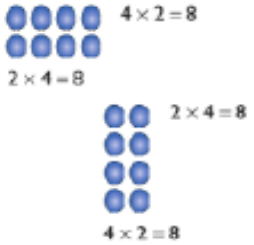
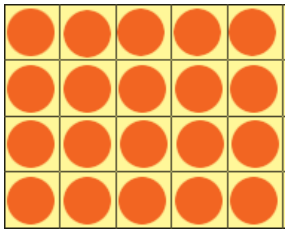
Draw arrays in different rotations to find **commutative** multiplication sentences.





Link arrays to area of rectangles.

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Use an array to write multiplication sentences and reinforce repeated addition.

5 + 5 + 5 = 15

3 + 3 + 3 + 3 + 3 = 15

5 x 3 = 15

3 x 5 = 15

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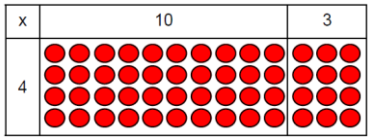
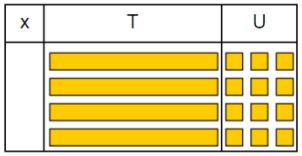
Grid Method

Show the link with arrays to first introduce the grid method.



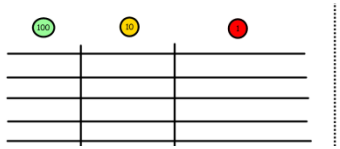
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.



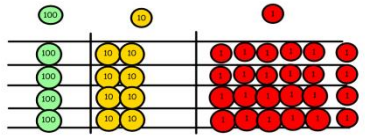
4 rows of 13

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



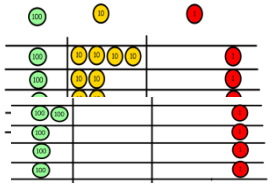
Calculations
4 x 126

Fill each row with 126.



Calculations
4 x 126

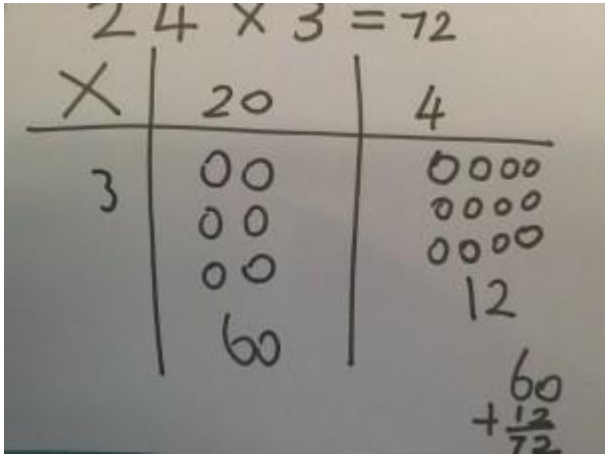
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

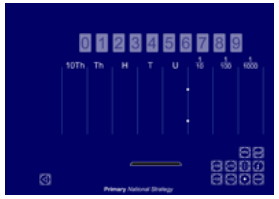
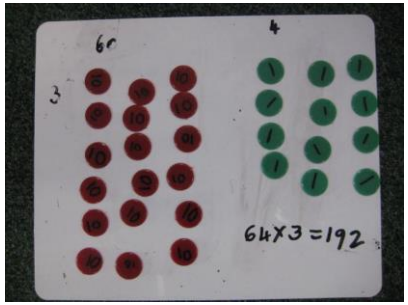
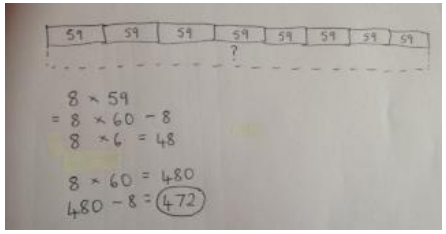
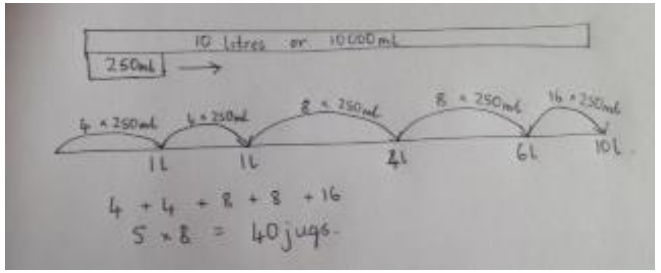
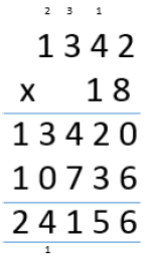
×	30	5
7	210	35

210 + 35 = 245

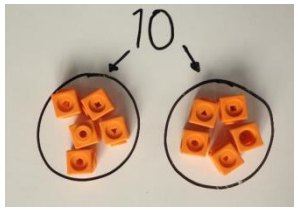
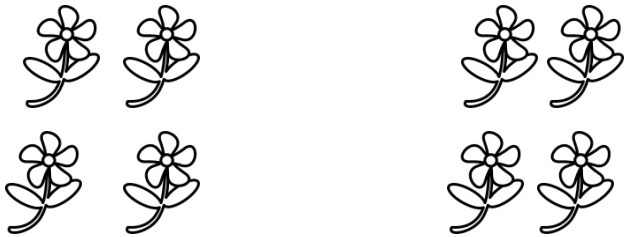
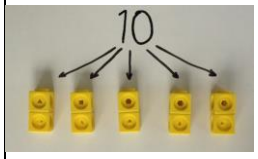
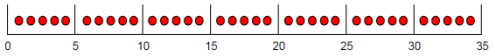
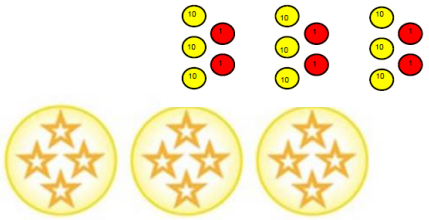
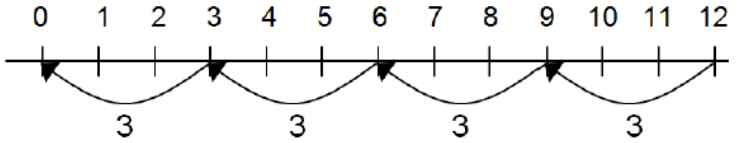
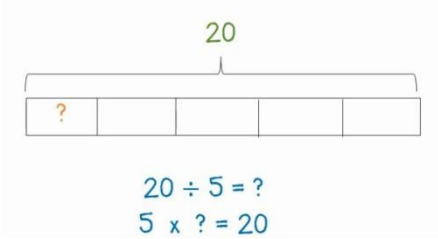
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

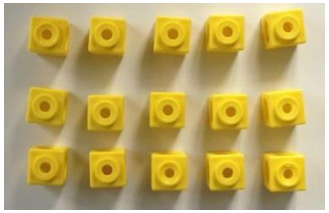
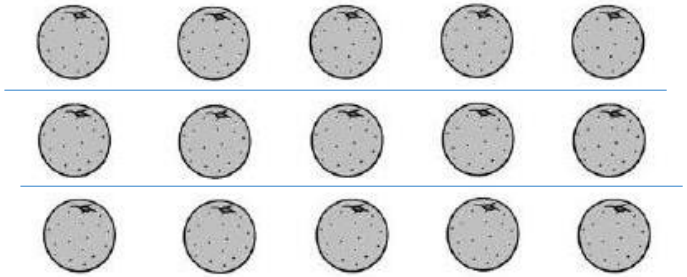
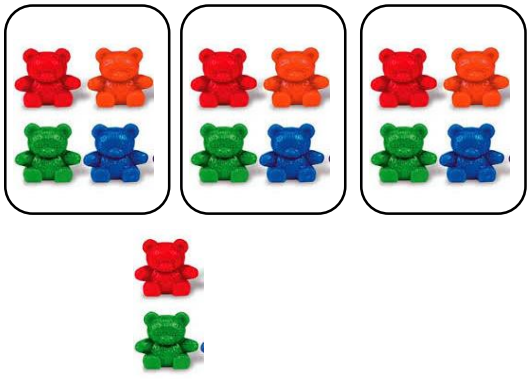
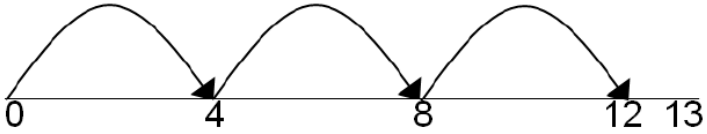
	10	8
10	100	80
3	30	24

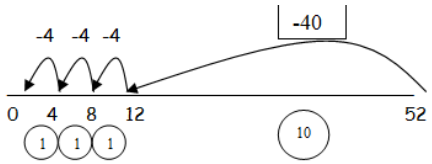
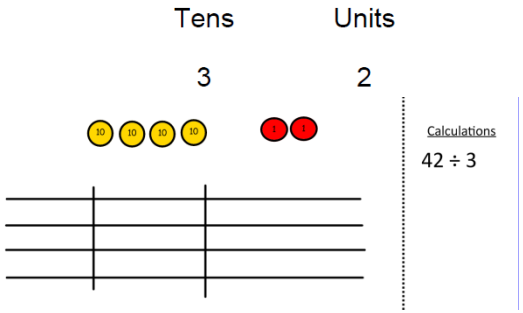
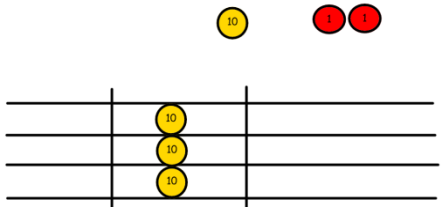
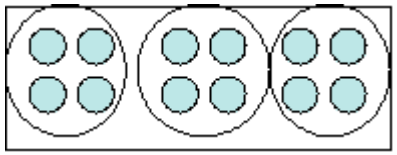
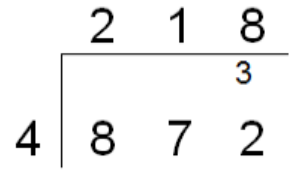
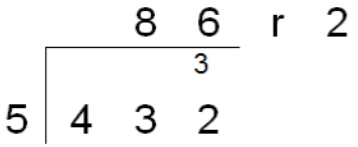
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

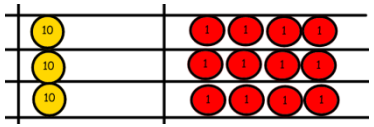
Multiplying by 10, 100, 1000		Use the Place Value grids to see what happens when numbers get 10x bigger.	
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Children use known facts to help them with the calculations: if 5 x 5 = 25, then 8 x 5 = 25 + 5+5+5=40. Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. Children could write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768  This moves to the more compact method. 

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups Sharing = (divided **between**)	 I have 10 cubes, can you share them equally in 2 groups? 10 ÷ 2= 5 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping Grouping (divided **by**) initially linked to repeated subtraction dividend ÷ divisor = quotient	Divide quantities into equal groups of the divisor: 10 ÷ 2= 5 Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups 12 ÷ 3= 4  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15 **Division is not commutative i.e. 15 ÷ 3 is not the same as 3 ÷ 15**	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  	Complete written divisions and show the remainder using r. 

Division using 'Chunking' (see Appendix)		Continue to develop division as repeated subtraction. Encourage children at this stage to move in larger jumps/steps. e.g. x10 of number being divided. **e.g. 52 ÷ 4= 13** Children write a bank of known facts usiing doubling, halving and multiplying by 10: 1 → 4 2 → 8 4 → 16 10 → 40 5 → 20  This method should still be used to answer calculations involving remainders.	**Long Division/Chunking** Children should now move onto vertical methods, starting with long division. **e.g.52÷4=13** 13 4) 52 - 40 (10x) 12 - 4 (1x) 8 - 4 (1x) 4 - 4 (1x) 0 **N.B.** Some children may recognise 12 is 3 groups of 4 and not subtract 1 group of 4 at a time. **Answer 13**
Short division	 Use place value counters to divide using the bus stop method alongside 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over. 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder. 

	We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.		Finally move into decimal places to divide the total accurately.